OSCILLATORY CONVECTIVE INSTABILITY OF THE EQUILIBRIUM OF TWO-LAYER SYSTEMS IN THE PRESENCE OF A THERMOCAPILLARY EFFECT

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It is well known that in a homogeneous liquid the convective instability of equilibrium has a monotonic character [1]. The possibility of a thermocapillary oscillatory instability for systems of two infinitely thick layers was established in [2]. For layers with finite thickness the oscillatory instability of equilibrium has been observed for both thermocapillary [3, 4] and thermogravitational convection [5]. In this paper we study the appearance of oscillations under the combined action of both instability mechanisms. It is established that the finite thickness of the layers substantially changes the criterion for the appearance of oscillatory convection, and the determining parameter is the ratio of the thickness of the layers. It is shown that under the combined action of thermocapillary and thermogravitational mechanisms of convection the oscillatory instability may turn out to be most dangerous even for systems for which the instability is monotonic in the presence of only one of the mechanisms.

1. Let the space between two solid horizontal plates, on which a constant and different temperature is maintained (the temperature difference is equal to θ), be filled with two layers of immiscible viscous liquids. The x axis is oriented horizontally and the y axis is oriented vertically upwards. The equations of the solid boundaries are $y = a_1$ and $y = -a_2$. The coefficients of dynamic and kinematic viscosity, thermal conductivity, thermal diffusivity, and volume expansion are η_m, ν_m, $\varkappa_m$, χ_m, β_m (m = 1 for the upper liquid and m = 2 for the lower liquid), respectively. The coefficient of surface tension depends linearly on the temperature: $\sigma = \sigma_0 - \alpha T$.

It is well known that the curvature of the interface is significantly only for long-wavelength perturbations [6]. In this paper, such perturbations are not studied and the interface is assumed to be flat (y = 0).

We introduce the following notation: $\eta = \eta_1/\eta_2$, $\nu = \nu_1/\nu_2$, $\varkappa = \varkappa_1/\varkappa_2$, $\beta = \beta_1/\beta_2$, $\chi = \chi_1/\chi_2$, $a = a_2/a_1$. We choose the quantities a_1, a_1^2/ν_1, ν_1, and θ as the units of length, time, stream function, and temperature, respectively. The dimensionless temperature gradient dT_0/dy in equilibrium is equal to $A_1 = -s/(1 + \varkappa a)$ in the upper liquid and $A_2 = -s\varkappa/(1 + \varkappa a)$ in the lower liquid, where s = −1 for heating from above and s = 1 for heating from below. For normal perturbations, the stream function ψ_m and the temperature T_m (m = 1, 2) with wave number k and complex increment $\lambda + i\omega$, the linearized equations of convection have the form

$$(\lambda + i\omega)D\psi_m = -d_m D^2\psi_m + ik\mathrm{Gr}\, b_m T_m,$$
$$-(\lambda + i\omega)T_m - ik\psi_m A_m = \frac{c_m}{\mathrm{Pr}} DT_m, \qquad (1.1)$$

where $D = d^2/dy^2 - k^2$, $d_1 = b_1 = c_1 = 1$, $d_2 = 1/\nu$, $b_2 = 1/\beta$, $c_2 = 1/\chi$, $\mathrm{Pr} = \nu_1/\chi_1$ is Prandtl's number, and $\mathrm{Gr} = g\beta_1\theta a_1^3/\nu_1^2$ is the Grashof number.

The conditions on the solid boundaries are

$$y = 1:\ \psi_1 = \psi_1' = T_1 = 0,\ y = -a:\ \psi_2 = \psi_2' = T_2 = 0; \qquad (1.2)$$

and the conditions at the interface are

$$y = 0:\ \psi_1 = \psi_2 = 0,\quad \psi_1' = \psi_2',\quad T_1 = T_2,\quad \varkappa T_1' = T_2',$$
$$\eta\psi_1'' - ik\,\mathrm{Mr}\,T_1 = \psi_2'', \qquad (1.3)$$

$\mathrm{Mr} = \eta M/\mathrm{Pr}$ ($M = \alpha\theta a_1/\eta_1\chi_1$) is the Marangoni number. The boundary of stability of equilibrium is determined by the condition $\lambda = 0$.

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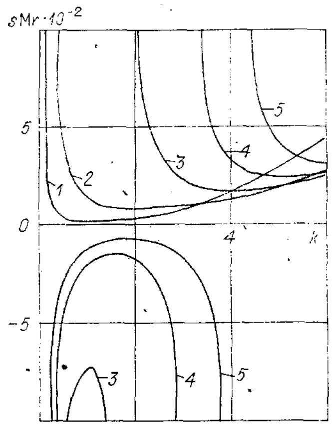

Fig. 1

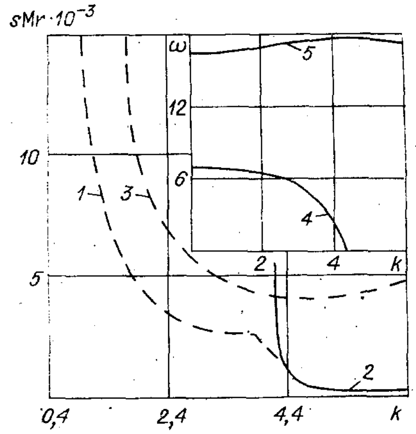

Fig. 2

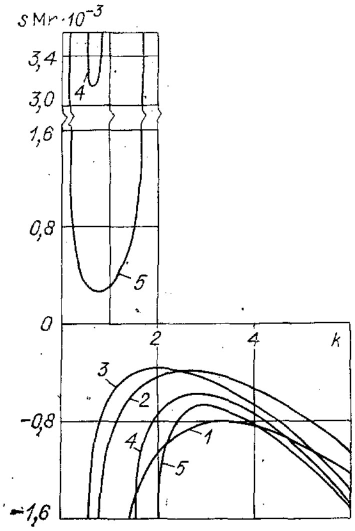

Fig. 3

The solution of the boundary-value problem (1.1)-(1.3) can be obtained in a simple analytic form only in the case of the monotonic instability ($\omega = 0$) with Gr = 0 [7]. In this work, in order to obtain the limits of stability, the solution of the boundary-value problem was constructed by the Runge–Kutta method.

2. The convective stability of two-layer systems in the presence of a thermocapillary effect was studied in [2, 6-9]. In [6-9] the monotonic instability of the equilibrium was studied. In [2] the possibility of the appearance of an oscillatory instability for infinitely thick layers, which corresponds physically to the case when the thickness of the layer is much greater than the wavelength of the perturbations, was also studied. In a real situation (for layers of finite thickness). However, the most dangerous perturbations are those whose wavelengths are of the order of the thicknesses of the layers, the results in [2] describe only the short-wavelength asymptotic behavior of the neutral curves. The ratio of the thicknesses of the layers is one of the determining parameter.

As an example, we shall present the results of a calculation of the neutral curves, performed for a system consisting of formic acid and transformer oil with the following parameters: Pr = 14.2, $\nu = 0.065$, $\eta = 0.09$, $\chi = 1.40$, $\varkappa = 2.44$ (there is no gravity and Gr = 0). According to the criteria presented in [2], in the case of infinitely thick layers, for the given system only the monotonic instability can occur, and only for one direction of the temperature gradient; for the opposite orientation of the temperature gradient, the equilibrium is absolutely stable. As calculations show, the results are substantially different for layers of finite thickness. The monotonic instability is realized only with heating from the side of the transformer oil and only for $a > a_*$ (it can be shown that $a_* = 1/\sqrt{\chi} = 0.85$ [7]). For $a < a_*$, an interval of wave numbers in which the monotonic instability appears with the opposite method of heating, appears in the long-wavelength region. The neutral curves are shown in Fig. 1: the lines 1-5 correspond to $a = 2$, 1, 0.8, 0.7, and 0.6. In addition, an oscillatory instability appears in the long-wavelength region. Figure 2 shows the results of the calculation of the neutral curves sMr (the curves 1 and 3) and of the frequency (the curves of 4 and 5) for the oscillatory instability with $a = 0.6$ and 0.34. The curve 2 corresponds to the monotonic instability for $a = 0.6$.

3. The action of thermocapillary and thermogravitational mechanisms opens up new possibilities for the appearance of the oscillatory instability. We shall study a system consisting of water and No. 200 silicon oil with the following parameters: Pr = 6.28, $\nu = 1.116$, $\eta = 0.915$, $\chi = 0.472$, $\varkappa = 0.169$, $\beta = 7.16$. The results of the calculation of the neutral curves for the case of purely thermocapillary convection are shown in Fig. 3; the curves 1-5 correspond to $a = 0.2$, 0.4, 1, 1.6, and 2.4. The oscillatory instability was not observed for this system.

We fix the value $a = 1.6$ and study the combined effect of thermocapillary and thermogravitational mechanisms of convection. In the case when the thermocapillary mechanism is absent (Mr = 0) and heating occurs from below, a monotonic thermogravitational instability appears; several neutral curves corresponding to different modes are shown in Fig. 4a (lines

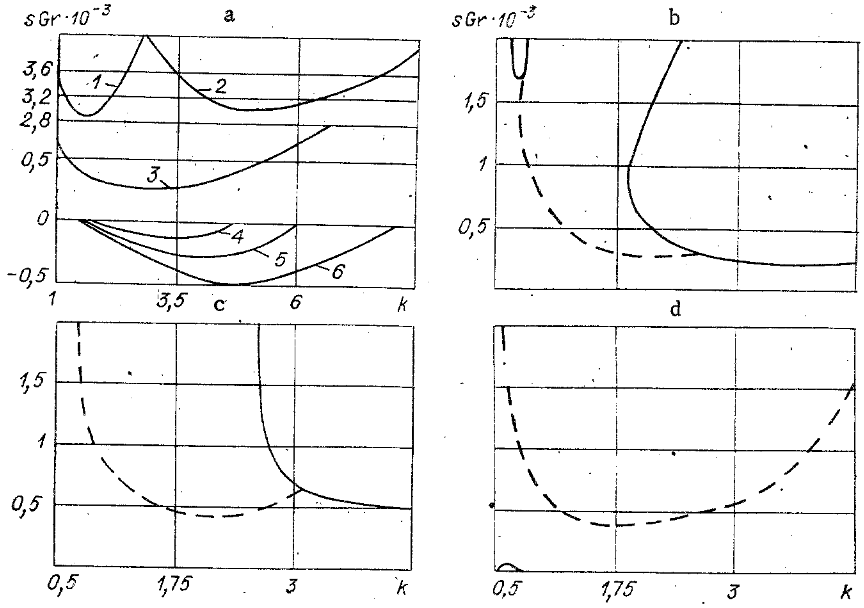

Fig. 4

1-3). The inclusion of the thermocapillary mechanism leads to the appearance of a section of oscillatory instability near the points of intersection of the neutral curves (Fig. 4b-d, sMr = 250, 500, and 3500, respectively, the sections of the oscillatory instability are shown by the broken line, and the sections of the monotonic instability are shown by the solid line). The oscillatory instability appeared in an analogous manner when the parameter a was changed in the case of the purely thermogravitational convection [5]. In contrast to the case studied in [5], as sMr is increased the oscillatory perturbations become most dangerous (Fig. 4c).

We note that when gravity appears, the thermocapillary instability (corresponding to the line 4 in Fig. 3) stabilizes for both heating from below (sMr = 3500, Fig. 4d) and from above (lines 4-6, Fig. 4a corresponded to sMr = −1000, −1500, −2000), as Gr increases, the interval of wave numbers in which instability appears narrows and vanishes.

Thus, in a real situation it may be expected that oscillatory states of convection will appear even in cases when the criteria obtained in [2] predict absolute stability or monotonic instability of equilibrium.

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